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# The KM phase in semi-realistic heterotic orbifold models

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## Abstract

In string-inspired semi-realistic heterotic orbifolds models with an anomalous  $U(1)_X$ , a nonzero Kobayashi-Masakawa (KM) phase is shown to arise generically from the expectation values of complex scalar fields, which appear in nonrenormalizable quark mass couplings. Modular covariant nonrenormalizable superpotential couplings are constructed. A toy  $Z_3$  orbifold model is analyzed in some detail. Modular symmetries and orbifold selection rules are taken into account and do not lead to a cancellation of the KM phase. We also discuss attempts to obtain the KM phase solely from renormalizable interactions.

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# 1 Introduction

Numerous attempts have been made to explain the origin of CP violation in the context of string-derived effective supergravity models. Early on, Strominger and Witten suggested compactification might lead to *explicit* violation of CP from the four-dimensional point of view, since the operation of CP is orientation-changing in the six-dimensional compact space [1]. Inspired by this prospect, detailed analyses were carried out for various compactifications of the heterotic string; for example, a Calabi-Yau manifold was examined in [2] and  $Z_N$  orbifolds were investigated in [3]. In each case where explicit CP violation was investigated, it was found not to occur. Subsequently, Dine et al. argued that CP is a gauge symmetry in string theory and that explicit breaking is therefore forbidden both perturbatively and nonperturbatively [4]. In the same work it was shown that this is certainly true for heterotic orbifolds. Based on these results, it is clear that if a heterotic orbifold is to provide a reasonable approximation to the correct underlying theory of known interactions, CP violation must occur *spontaneously* from string moduli or matter fields (or perhaps both) acquiring vacuum expectation values (*vevs*).

In a series of papers, Bailin, Kraniotis and Love (BKL) considered the possibility of supersymmetric CP violation (SCPV)<sup>1</sup> in heterotic orbifold models [6]. They related SCPV to the complex phases of string moduli vevs. While SCPV is an interesting possibility, it is important to understand how the Kobayashi-Masakawa (KM) phase [7] might occur in semi-realistic orbifold models and how generic it is. Furthermore, SCPV in the context of supergravity is phenomenologically problematic unless soft terms meet a variety of stringent criteria [5, 8]; the current status of SCPV and various solutions to related phenomenological problems are reviewed in ref. [9]. By contrast, the KM phase does not have dangerous side-effects; the smallness of CP violation is for the most part explained by the small mixing angles between heavy and light generation quarks while large electric dipole moments do not arise from this source [10].

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<sup>1</sup>By this, we mean CP-violating complex phases in the soft supersymmetry breaking operators of minimal extensions to the Standard Model (SM) [5].

BKL have considered complex vevs for the string moduli of heterotic orbifolds as a source of the KM phase [11].  $T$ - and  $U$ -moduli, which parameterize deformations of the six-dimensional compact space consistent with the orbifold construction, enter into the effective Yukawa couplings for twisted fields, as described below. In this article we will restrict ourselves to  $Z_3$  and  $Z_3 \times Z_3$  orbifolds, which have no U-moduli because the complex structure is fixed. It has been demonstrated by BKL and others that is possible for non-perturbative effects to stabilize these string moduli at complex values [6, 12]. BKL have shown that, as a result of complex string moduli vevs,  $\mathcal{O}(1)$  phases can arise in twisted Yukawa coupling coefficients. However, they do not construct any models and it is unclear whether or not the phases they find are physical: phases in the quark Yukawa matrices do not necessarily imply the existence of a nonzero KM phase.<sup>2</sup> As an example, let us write the quark mass superpotential as

$$W_{qm} = \lambda_{ij}^u H_u Q_{iL} u_{jL}^c + \lambda_{ij}^d H_d Q_{iL} d_{jL}^c \quad (1.1)$$

and suppose Yukawa matrices of the form

$$\lambda_{ij}^u = |\lambda_{ij}^u| e^{i(\alpha_i + \beta_j^u)}, \quad \lambda_{ij}^d = |\lambda_{ij}^d| e^{i(\alpha_i + \beta_j^d)}. \quad (1.2)$$

The phases  $\alpha_i, \beta_j^{u,d}$  are not physical since they can be removed by rephasing the quark fields according to

$$Q_{iL} \rightarrow e^{-i\alpha_i} Q_{iL}, \quad u_{jL}^c \rightarrow e^{-i\beta_j^u} u_{jL}^c, \quad d_{jL}^c \rightarrow e^{-i\beta_j^d} d_{jL}^c. \quad (1.3)$$

Naive orbifold models of quark Yukawa couplings assign the nine SM multiplets  $Q_{iL}, u_{jL}^c, d_{jL}^c$  to different sectors of the Hilbert space and rely on trilinear couplings to give all of the quarks mass; the assignments to different sectors determine the moduli-dependence of the effective Yukawa matrices, hence the complex phases. If these assignments can be brought into correspondence with phases that enter the Yukawa matrices in a way similar to (1.2), then they will not give rise to CP violation. An example of this is described in Section 2 below. Furthermore, when one reviews the tables of phases displayed in the appendix of ref. [11], one finds that many of them are identical

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<sup>2</sup>Complex phases which give rise to a nontrivial KM phase will be termed *physical* while those which can be eliminated by rephasing quark fields will be termed *spurious*.

or zero for a given orbifold model and T-modulus vev. It then becomes a concern whether or not this high degree of degeneracy causes the phases to “wash out” in the final analysis. An example of this is also given in Section 2.

Beyond these more obvious ways that complex phases in the Yukawa matrices may not give rise to a nontrivial KM phase, we must also be concerned that symmetry constraints imposed on the Yukawa matrices by the underlying string theory might relate the phases to each other in such a way that the CKM matrix can be made entirely real. The embedding of the orbifold action into the internal left-moving gauge degrees of freedom typically leaves a surviving gauge group significantly larger than the SM. For example, in the three generation constructions which we will discuss below the surviving gauge group  $G$  has rank sixteen [13, 14]. Low-energy effective Yukawa couplings must be constructed from high-energy operators invariant under  $G$ . The high-energy operators are also subject to orbifold selection rules, which result from symmetries of the six-dimensional compact space. (A brief review of orbifold selection rules may be found in ref. [14].) Finally, the underlying conformal field theory has target space modular symmetries associated with the identification of equivalent string moduli backgrounds. It is conceivable (albeit highly unlikely) that orbifold selection rules, gauge invariance under  $G$  and target-space modular invariance may conspire to make phases derived from the scalar background spurious. In the present paper we construct a toy  $Z_3$  orbifold model which is subject to these symmetry constraints; we construct explicit quark mass matrices in order to make conclusive statements about the existence of a nonzero KM phase.

The case of complex vevs for the T-moduli is treated below; however, we place more emphasis on another origin of complex phases, which in our opinion is a much more generic and natural source of CP violation in semi-realistic heterotic orbifold models. Nonrenormalizable couplings are often important in semi-realistic heterotic orbifold models for the following reason. Wilson lines are typically included in the embedding to get a reasonable gauge group.<sup>3</sup> In many cases their inclusion leads to an anomalous  $U(1)_X$

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<sup>3</sup> Standard GUT scenarios require large higgs representations, whereas they are absent in affine level 1 orbifold constructions [15]. This makes it phenomenologically advantageous to obtain  $G = G_{SM} \times G_{other}$  from the start.

factor:  $\text{tr } Q_X \neq 0$ . The apparent anomaly is cancelled by the Green-Schwarz mechanism, which induces a Fayet-Iliopoulos (FI) term [16]. Several scalar fields get vevs  $v_i \sim \mathcal{O}(10^{-2\pm 1})$  (in units where  $m_P = 1/\sqrt{8\pi G} = 1$ ) to restabilize the vacuum in the presence of the FI term; thus, the suppression of nonrenormalizable couplings due to vevs  $v_i$  may be as little as  $\mathcal{O}(10^{-1})$ . The scalar fields which cancel the FI term break the  $U(1)_X$  gauge symmetry at the scale  $\Lambda_X \sim 10^{17}$  GeV by the Higgs mechanism; in order to distinguish them from the higgses associated with electroweak symmetry breaking, we will for convenience and with all due apologies refer to them as *Xiggses*. The Xiggses are usually charged under other  $U(1)$  factors of  $G$  besides  $U(1)_X$ ; a basis of generators can always be chosen such that these other  $U(1)$ 's are not anomalous.<sup>4</sup> Vacuum stabilization near the scale  $\Lambda_X$  requires the D-terms of these other  $U(1)$ 's to vanish. To satisfy the numerous D-flatness conditions, it is generally necessary for several Xiggses to get vevs. Typically there are  $\mathcal{O}(10)$  or more such Xiggses.

In this paper we demonstrate how the KM phase in semi-realistic heterotic orbifold models arises generically from the “Planck slop” created by the Xiggses. Nonrenormalizable couplings make significant and in some cases leading order contributions to the effective quark mass matrices. For instance, in the semi-realistic model developed by Font, Ibáñez, Quevedo and Sierra in Section 4.2 of ref. [14] (*FIQS model*), the down-type quarks get their leading order mass from dimension 10 holomorphic couplings. (We count dimensions as 1 for each elementary superfield entering a coupling.) For up-type quarks, only the top and charm get masses from renormalizable couplings.<sup>5</sup> The up quark must receive its mass from nonrenormalizable couplings or radiative mass terms. Although high order nonrenormalizable couplings are suppressed by large powers of the  $\mathcal{O}(10^{-2\pm 1})$  Xiggs vevs, the suppression is not as large as one might think, for two reasons. Many high order operators involving the Xiggses generically exist, with their number increasing at each higher order. Most of the operators are closely related by variations in

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<sup>4</sup>We will not consider the case where Xiggses are in nontrivial representations of the nonabelian factors of  $G$ .

<sup>5</sup>We identify top, charm, etc., by ranking the mass at a given level of analysis; of course, the identification will be imperfect once higher-order corrections are included.

fixed point locations and oscillator “directions”, as will be shown in detail below; however, this does not change the fact that the number of distinct operators tends to be large. The second reason why high order couplings are important is that the coupling strength tends to be much larger than  $\mathcal{O}(1)$  and to grow as the dimension of the coupling increases, as was pointed out by Cvetič et al. [17]. The combination of these two effects forces one to proceed to rather high order before couplings make negligible contributions to the quark mass matrices. Here, “negligible” is taken to mean less than, say, 10% contributions to the lightest quark masses. As a consequence, each effective Yukawa coupling  $\lambda_{ij}^{u,d}$  depends on the vev of a linear combination of a large number of monomials of Xiggses. The principal point of this article is that since  $\mathcal{O}(10)$  Xiggses get complex vevs, which appear in a large number of monomials contributing to effective quark Yukawa couplings, the Yukawa matrices are generically complex and a nonvanishing KM phase is almost inevitable. Indeed, in any orbifold model with an anomalous  $U(1)_X$  present, it seems improbable that one would not have CP violation in this way, since it is difficult to see how nonrenormalizable couplings involving the Xiggses would not contribute to the effective quark Yukawa couplings.

Previous authors have noted the possible role of nonrenormalizable couplings in heterotic orbifold and free fermionic models for giving large mass hierarchies and CP violation from a KM phase [18, 19, 20]. In this respect the mechanism analyzed here is not new. However, we present much more detailed results by constructing an explicit toy model and we impose modular invariance on the nonrenormalizable couplings. The constraint of modular invariance leads to relationships between coupling coefficients which were not accounted for in earlier efforts. The model is inspired by three generation heterotic  $Z_3$  orbifold models previously investigated in the literature [13, 14]. Our model is, by construction, quite similar to the FIQS model, which is string-derived: in the FIQS model, the gauge group, the spectrum of states and the allowed superpotential couplings are completely determined from the underlying string theory. The FIQS model suffers from phenomenological difficulties related to the quark mass matrices; these difficulties were previously pointed out in [21]. At leading order in the FIQS model, the top and charm come from different  $SU(2)$  doublets than the bottom. As a result, the lead-

ing order CKM matrix has some of its diagonal entries zero and some  $\mathcal{O}(1)$  heavy-light generation mixing angles, which is clearly unacceptable. The assignments into  $SU(2)$  doublets are determined by the H-momenta<sup>6</sup> of the untwisted states, so the identification of which doublet a given  $u_{iL}$  or  $d_{iL}$  sits in and how it couples at leading order is fixed. A second problem with the FIQS model is that it is difficult to give the three lightest quarks mass. We have searched for dimension  $d \leq 30$  allowed holomorphic couplings which might do the job (under the assumptions made in the FIQS model about Xiggs and T-moduli vevs) and found that none exist. We further found that radiative masses only occur at high loop levels and would be minuscule in comparison to the experimental values. However, we assumed the leading order Kähler potential in this analysis. It is possible that higher order terms in the Kähler potential could provide a mechanism for giving the light quarks masses in agreement with experimental values.

We have attempted in many ways to evade these problems. However, we were not able to do so without creating other difficulties. We are actively searching for a superior three generation heterotic  $Z_3$  orbifold model to study. In the meantime, we have developed a toy model which replicates the FIQS model wherever possible while avoiding its problems, in order that we might illustrate that a nontrivial KM phase generically arises from the complex vevs of Xiggses, even after orbifold selection rules and target space modular invariance have been accounted for.

Although the coupling coefficients for nonrenormalizable superpotential couplings (which are in principle obtainable from the underlying conformal field theory) are not apparently known, we propose couplings which transform in the requisite manner under the  $[SL(2, \mathbf{Z})]^3$  diagonal subgroup of the full  $SU(3, 3, \mathbf{Z})$  modular duality group of the  $Z_3$  orbifold [22]. We take into account the non-trivial transformations of twisted sector fields in our construction of modular invariant couplings. In the three generation  $Z_3$  orbifold models upon which the toy model of Section 4 is based, different *species* are distinguished by quantum numbers (other than the fixed point location in the third complex plane) of massless states in the underlying theory [14, 13].

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<sup>6</sup> H-momenta are the  $SO(10)$  weights of bosonized NSR fermions, which appear in the vertex operators creating asymptotic states.



For twisted fields  $\Phi_n^i$ , where  $n$  labels the species and  $i = 1, 2, 3$  labels the fixed point locations in the third complex plane,  $\Phi_n^1, \Phi_n^2$  and  $\Phi_n^3$  mix amongst themselves under the  $T^3 \rightarrow 1/T^3$  duality transformation in the third complex plane [23, 24]. Constraining holomorphic polynomials of dimension  $d > 3$  to transform with<sup>7</sup> modular weight  $-1$  in light of these nontrivial mixings places strong constraints on the form of the superpotential terms and gives some confidence that our proposed nonrenormalizable coupling coefficients may reproduce key features of the actual couplings which would be derived using conformal field theory techniques. In the course of discussing our assumptions for the coupling coefficients of nonrenormalizable superpotential terms, we will explain why the calculation of these coefficients from the underlying string theory represents an extremely difficult problem. For now, we remark that the most intimidating aspect of such a calculation is the integration of the string correlator over the  $d - 3$  vertex locations which cannot be fixed by  $SL(2, \mathbf{C})$  invariance, where  $d$  is the dimension of the coupling. The string correlator is generally a very complicated function of the unfixed vertex locations.

The reliability of the effective supergravity approach is hampered by theoretical uncertainties in the Kähler potential of  $Z_3$  orbifold models. Nonleading operators in the Kähler potential are neglected in most analyses; however, with  $\mathcal{O}(10^{-2\pm 1})$  Xiggs vevs, higher order terms in the Kähler potential give non-negligible corrections to the mass matrices of quarks: corrections from higher order terms give the quarks noncanonical, nondiagonal kinetic terms which must be rendered canonical by nonunitary field redefinitions when one goes to compute mass eigenstates and mixings. These complications cannot be ignored if one hopes to develop an accurate picture of the low-energy phenomenology predicted by a given model. Higher order terms in the Kähler potential ought to be included in order to be consistent with the high order expansion of the superpotential, both of which are necessary in order to pick up all significant contributions to the quark mass matrices. The calculation of higher order corrections to the Kähler potential is notoriously difficult because of the lack of holomorphicity. It is hoped that future work on the Kähler potential of heterotic  $Z_3$  orbifold models will amend these deficiencies and

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<sup>7</sup>Modular weight will be explained below.

allow for an improved analysis of the low energy phenomenology of semi-realistic models. Present ignorance regarding these aspects of the Kähler potential has forced us to make a number of oversimplifications. However, we do not expect these oversimplifications to affect our main result that a nontrivial KM phase is generic. Introducing higher order terms in the Kähler potential is not expected to eliminate the complex phases which enter into the effective quark Yukawa couplings from the vevs of Xiggses.

In Section 2 we present examples where the complex phases found by BKL are spurious. In Section 3 we introduce modular invariant coupling coefficients for nonrenormalizable superpotential couplings. In Section 4 we discuss our string-inspired toy heterotic  $Z_3$  orbifold model. In Section 5 we make concluding remarks and suggest further investigations motivated by our results. In the Appendix we address normalization conventions for  $U(1)$  charges in string-derived models. We explain how to account for different conventions when determining the FI term and describe the Green-Schwarz cancellation of the  $U(1)_X$  anomaly in the linear multiplet formulation.

## 2 Counterexample

Here, we consider some assignments of quarks and higgses in  $Z_3 \times Z_3$  orbifold models and show that the complex phases found by BKL do not lead to a nontrivial KM phase for these particular examples. We certainly do not wish to imply that the phases found by these authors cannot lead to a nonzero KM phase; we only wish to point out that due to the degeneracy in phases (in this case always 0 or  $-\pi/3$ ), they can in many cases be eliminated by rephasing quark fields. In our opinion, a more careful analysis is required in order to conclude whether or not the phases found by BKL can account for CP violation.

We will use the notation and conventions of ref. [25] in our discussion of the twisted sectors and fixed tori of the  $Z_3 \times Z_3$  orbifold. This orbifold is constructed using twists

$$\theta = \frac{1}{3}(1, 0, -1), \quad \omega = \frac{1}{3}(0, 1, -1). \quad (2.1)$$

We make use of the  $\theta$ ,  $\theta\omega$  and  $\theta\omega^2$  twisted sectors. The fixed tori for each of

these sectors are given by

$$f_\theta = \frac{m_1}{3}(2e_1 + \tilde{e}_1) + \frac{m_3}{3}(e_3 - \tilde{e}_3) + v_2, \quad m_{1,3} = 0, \pm 1, \quad v_2 \in K_2, \quad (2.2)$$

$$f_{\theta\omega} = \frac{1}{3} \sum_{i=1}^3 r_i(2e_i + \tilde{e}_i) + \ell, \quad r_i = 0, \pm 1, \quad \ell \in \Lambda, \quad (2.3)$$

$$f_{\theta\omega^2} = \frac{p_1}{3}(2e_1 + \tilde{e}_1) + \frac{p_2}{3}(e_2 - \tilde{e}_2) + v_3, \quad p_{1,2} = 0, \pm 1, \quad v_3 \in K_3, \quad (2.4)$$

where  $\Lambda$  is the  $[SU(3)]^3$  root lattice and  $K_i$  is the  $i$ th complex plane. Physical states must be simultaneous eigenstates of  $\theta$  and  $\omega$ ; they are therefore linear combinations of states whose zero modes are given by different fixed tori. Since the first complex plane is neutral under  $\omega$ , physical states in the  $\theta$  sector can be chosen with a definite quantum number  $m_1$ . Because the form of the fixed torus (2.4) in the first complex plane is the same as in the  $\theta$  sector (2.2), and since  $\omega$  does not rotate in the first plane, a physical state in the  $\theta\omega^2$  sector can be chosen to have a definite quantum number  $p_1$  as well. The coefficients of the trilinear Yukawa couplings are determined by the evaluation of correlation functions in the underlying string theory. The contribution from a complex  $T^1$  in the classical partition function is the only source of complex phases in the trilinear couplings. The phases of  $T^{2,3}$  do not matter because the twist operator contributions to the correlation function in the second and third complex planes reduce to the identity, as discussed in ref. [25]. The phases found in ref. [11] for the case  $\langle T^1 \rangle = \exp(i\pi/6)$  are determined by the difference  $m_1 - p_1$ :

$$\gamma(m_1, p_1) \equiv \arg \sum_{X_{cl}} e^{-S_{cl}} = \begin{cases} 0, & m_1 - p_1 = 0; \\ -\frac{\pi}{3}, & m_1 - p_1 = \pm 1, \pm 2. \end{cases} \quad (2.5)$$

We next suppose assignments as follows:  $H_u, H_d$  are in the  $\theta$  sector with fixed tori quantum numbers  $m_1^u, m_1^d$  resp.;  $Q_{iL}$  are in the  $\theta\omega^2$  sector with fixed tori quantum numbers  $p_1^i$  resp.;  $u_{jL}^c, d_{jL}^c$  are in the  $\theta\omega$  sector. The phases (2.5) enter into the Yukawa couplings (1.1) according to:

$$\lambda_{ij}^u = |\lambda_{ij}^u| e^{i\alpha_i^u}, \quad \lambda_{ij}^d = |\lambda_{ij}^d| e^{i\alpha_i^d}, \quad (2.6)$$

where  $\alpha_i^u = \gamma(m_1^u, p_1^i)$  and  $\alpha_i^d = \gamma(m_1^d, p_1^i)$ . If  $m_1^u = m_1^d$  then we recover the CP conserving forms of (1.2) with  $\alpha_i = \alpha_i^u = \alpha_i^d$  and  $\beta_j^{u,d} = 0$ .

It is an interesting question whether or not CP violation really can occur, and the example just presented demonstrates that a careful case-by-case analysis is probably required in order to draw any firm conclusions. It should be noted that in the case where nonrenormalizable couplings give significant corrections to the trilinear  $(\theta, \theta\omega, \theta\omega^2)$  coupling, the above arguments likely fail.

Lastly, we would like to comment on another suggested source of complex phases. As noted above, physical states are constructed from linear combinations of states whose zero modes are given by different fixed tori. It was noted by Kobayashi and Ohtsubo that complex phases enter from the coefficients in the linear combinations and that these might be a source of CP violation [26]. However, this amounts to explicit CP violation since it does not require a particular scalar background. As explained in Section 1, explicit CP violation does not occur in heterotic orbifolds since CP is a gauge symmetry of the underlying theory. Therefore, the phases which enter into couplings from this source cannot contribute to the KM phase.

### 3 Modular invariance

In the toy model to be considered below, nonrenormalizable superpotential couplings will play a crucial role. In this section we present a set of assumptions for coupling coefficients of holomorphic couplings of arbitrary order; the result will be couplings which transform in the requisite manner under the  $[SL(2, \mathbf{Z})]^3$  subgroup of the full  $SU(3, 3, \mathbf{Z})$  modular duality group of the  $Z_3$  orbifold.

We begin by considering the simpler case of a two dimensional  $Z_3$  orbifold, where there is a single modulus  $T$  and there are only three fixed points. Consequently, twisted matter fields carry a single fixed point label. The twisted trilinear couplings are known in this simple case [27]. These couplings can be expressed in terms of the Dedekind  $\eta$  function

$$\eta(T) = e^{-\pi T/12} \prod_{n=1}^{\infty} (1 - e^{-2\pi n T}) \quad (3.1)$$

$T$	$\eta(T)$	$\chi_0(T)$	$\chi_1(T)$
1	0.76823	1.7134	0.97280
$e^{i\pi/6}$	$0.80058 e^{-i\pi/24}$	$1.5197 e^{i\pi/12}$	$0.91251 e^{-i\pi/12}$

Table 1: Values of modular functions at self-dual points.

and the level-one  $SU(3)$  characters [23]

$$\chi_i(T) = \eta^{-2}(T) \sum_{v \in \Gamma_i} e^{-\pi T |v|^2}. \quad (3.2)$$

In this expression,  $\Gamma_0$  is the  $SU(3)$  root lattice, while  $\Gamma_{1,2}$  are shifted by  $SU(3)$  weight vectors. Explicitly,

$$|v|^2 = \left( n_1 + \frac{i}{3}, \quad n_2 + \frac{i}{3} \right) \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} n_1 + i/3 \\ n_2 + i/3 \end{pmatrix} \quad (3.3)$$

with  $n_1$  and  $n_2$  integers to be summed over in (3.2) and  $i$  is the integer labeling  $\Gamma_i$ . It is easy to check that  $\chi_1 = \chi_2$ . The values of these three functions at the self-dual points  $T = 1, e^{i\pi/6}$  are approximately given by Table 1. The trilinear couplings between twisted fields  $\Phi_1^{i_1}, \Phi_2^{i_2}, \Phi_3^{i_3}$  in the superpotential are

$$\lambda \cdot [\eta(T)]^2 f_{i_1 i_2 i_3}^T \Phi_1^{i_1} \Phi_2^{i_2} \Phi_3^{i_3}, \quad (3.4)$$

where  $f_{i_1 i_2 i_3}^T$  is given by:

$$f_{i_1 i_2 i_3}^T = \begin{cases} \chi_0(T), & i_1 = i_2 = i_3; \\ \chi_1(T), & i_1 \neq i_2 \neq i_3 \neq i_1; \\ 0, & \text{else.} \end{cases} \quad (3.5)$$

The overall T-independent coupling strength  $\lambda$  does not depend on fixed point locations and is obtained by factorization of the four-point string correlator [27]. Note that  $\eta(T)$  is superfluous and occurs only because of the definition of  $\chi_i(T)$ .

It can be checked that the above Yukawa couplings transform correctly under the target-space modular duality group  $SL(2, \mathbf{Z})$ . The Kähler potential for the modulus  $T$  of the 2-d orbifold is given by  $K(T, \bar{T}) = -\ln(T + \bar{T})$ , which transforms under the  $SL(2, \mathbf{Z})$  modular transformations

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbf{Z}, \quad (3.6)$$

as

$$K(T, \bar{T}) \rightarrow K(T, \bar{T}) + F(T) + \bar{F}(\bar{T}), \quad (3.7)$$

where  $F(T) = \ln(icT + d)$ . We require that the quantity  $K + \ln|W|^2$  remain invariant [28], which implies that the superpotential  $W$  transform as

$$W \rightarrow e^{i\gamma(a,b,c,d)} e^{-F(T)} W, \quad (3.8)$$

where  $\gamma(a, b, c, d)$  is a  $T$ -independent phase which does not appear in the functional which is physically meaningful,  $K + \ln|W|^2$ . We can take  $b = -1, c = 1, d = 0$  to obtain the  $T \rightarrow 1/T$  transformation of  $SL(2, \mathbf{Z})$ , so that  $F(T) = \ln(iT)$  in this case. It has been shown [23, 24] that the twist fields  $\sigma_i$  which create twisted vacua corresponding to fixed points labeled by  $i$ , and hence the twisted states  $\Phi^i$ , transform under the duality transformation  $T \rightarrow 1/T$  as

$$\begin{pmatrix} \sigma'_1 \\ \sigma'_2 \\ \sigma'_3 \end{pmatrix} = \frac{e^{i\beta}}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \bar{\alpha} & \alpha \\ 1 & \alpha & \bar{\alpha} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}, \quad (3.9)$$

where  $\exp(3i\beta) \equiv \sqrt{\bar{T}/T}$  and  $\alpha \equiv \exp(2\pi i/3)$ . Since the vertex operator which creates the twisted state  $\Phi^i$  is proportional to  $\sigma_i$ , the twisted fields must transform in a way which is proportional to the same matrix. The nonholomorphic phase  $\beta$  must be absent in the supergravity definition of the fields. For example, we could define  $\tilde{\sigma}_i = e^{i\beta/2} \sigma_i$  and use these to create the supergravity fields, which must transform in a holomorphic way. Aside from the mixing of different fixed points, the twisted fields transform with a *modular weight*  $q$ :  $\Phi \rightarrow \Phi' \sim e^{-qF(T)} \Phi$ . These weights are known [29, 30, 31]; a non-oscillator twisted field has modular weight  $q = 2/3$ . Only non-oscillator twisted fields are allowed to enter into the trilinear twisted couplings due to the *automorphism selection rule*, corresponding to invariance of string

correlators under automorphisms of the underlying  $SU(3)$  lattice. This rule is explained and illustrated with examples in ref. [14]. Since  $F(T) = \ln(iT)$  for the  $T \rightarrow 1/T$  duality transformation, non-oscillator twisted fields must transform as

$$\begin{pmatrix} \Phi^{1'} \\ \Phi^{2'} \\ \Phi^{3'} \end{pmatrix} = \frac{1}{\sqrt{3}} \left( \frac{1}{iT} \right)^{2/3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \bar{\alpha} & \alpha \\ 1 & \alpha & \bar{\alpha} \end{pmatrix} \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \Phi^3 \end{pmatrix}. \quad (3.10)$$

Under the duality transformation  $T \rightarrow T' = 1/T$  it has been shown [23] that the  $SU(3)$  characters transform as

$$\begin{pmatrix} \chi'_0 \\ \chi'_1 \\ \chi'_2 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}. \quad (3.11)$$

It is also well-known that

$$\eta^2(1/T) = \eta^2(T') = T \eta^2(T) = -i e^{F(T)} \eta^2(T). \quad (3.12)$$

We define a polynomial  $p$  which encodes the superpotential couplings (3.4), up to a power of  $\eta(T)$ :

$$\begin{aligned} p(T; \Phi_1, \Phi_2, \Phi_3) &= \chi_0(T)(\Phi_1^1 \Phi_2^1 \Phi_3^1 + \Phi_1^2 \Phi_2^2 \Phi_3^2 + \Phi_1^3 \Phi_2^3 \Phi_3^3) \\ &\quad + \chi_1(T)(\Phi_1^1 \Phi_2^2 \Phi_3^3 + \Phi_1^3 \Phi_2^1 \Phi_3^2 + \Phi_1^2 \Phi_2^3 \Phi_3^1 \\ &\quad + \Phi_1^3 \Phi_2^2 \Phi_3^1 + \Phi_1^1 \Phi_2^3 \Phi_3^2 + \Phi_1^2 \Phi_2^1 \Phi_3^3). \end{aligned} \quad (3.13)$$

Using the transformation properties enumerated above, it can be shown that

$$\eta^2 p \rightarrow -i \left( \frac{1}{iT} \right) \eta^2 p = -i e^{-F(T)} \eta^2 p. \quad (3.14)$$

Thus, the functional  $\eta^2 p$  transforms with modular weight  $-1$  up to a moduli independent phase  $-i$ , as required by (3.8). Here, we draw attention to the fact that the monomials contained in (3.13) do not by themselves transform in the required way. Rather, it is the linear combination of fields in (3.13) together with  $\chi_i(T)$  factors which is modular covariant, in the sense of (3.8).

Similar arguments hold for the axionic shift  $T \rightarrow T' = T - i$ . Indeed,  $\eta^2(T) \rightarrow \exp(i\pi/6) \eta^2(T)$  and [23, 24]

$$\begin{pmatrix} \chi'_0 \\ \chi'_1 \\ \chi'_2 \end{pmatrix} = e^{-i\pi/6} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}, \quad (3.15)$$

$$\begin{pmatrix} \Phi^{1'} \\ \Phi^{2'} \\ \Phi^{3'} \end{pmatrix} = \begin{pmatrix} \bar{\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \Phi^3 \end{pmatrix}. \quad (3.16)$$

Then it can be checked that  $\eta^2 p \rightarrow \eta^2 p$ , which transforms as it should (E.g.,  $c = 0, d = 1$  for the axionic shift).

A general  $SL(2, \mathbf{Z})$  transformation (3.6) can be built up out of the two operations analyzed above. Thus, we are assured that the trilinear superpotential coupling (3.4) is modular covariant under the entire duality group. To summarize, for the general  $SL(2, \mathbf{Z})$  transformation the polynomial (3.13) and the Dedekind  $\eta$  function transform as

$$\begin{aligned} p(T; \Phi_1, \Phi_2, \Phi_3) &\rightarrow e^{i\phi(a,b,c,d)} e^{-\sum_{n=1}^3 q_n F(T)} p(T; \Phi_1, \Phi_2, \Phi_3), \\ \eta^2(T) &\rightarrow e^{i\gamma(a,b,c,d)} e^{F(T)} \eta^2(T), \end{aligned}$$

where  $q_n$  is the modular weight of the matter fields of species  $n$ ,  $\Phi_n^j$ . It can then be checked that (3.8) holds for the trilinear superpotential term coupling (3.4).

Twisted couplings of dimension  $3m > 3$  in the effective field theory remain to be calculated from the underlying conformal field theory. However, these computations appear formidable. As an example, we briefly consider the form of six dimensional twisted couplings using the methods of [27]. The classical action  $S_{cl}$  may be determined from *monodromy conditions* on the underlying bosonic fields  $X(z, \bar{z}), \bar{X}(z, \bar{z})$  of the 2-d orbifold, where  $z, \bar{z}$  provide a parameterization of the string world-sheet; for each classical solution  $X_{cl}(z, \bar{z})$ , *local* monodromy conditions demand

$$\begin{aligned} \partial_z X_{cl}(z) &= \frac{a(z_4, z_5, z_6)}{[z(z-1)(z-z_4)(z-z_5)(z-z_6)]^{2/3}}, \\ \bar{\partial}_{\bar{z}} X_{cl}(\bar{z}) &= \frac{b(\bar{z}_4, \bar{z}_5, \bar{z}_6)}{[\bar{z}(\bar{z}-1)(\bar{z}-\bar{z}_4)(\bar{z}-\bar{z}_5)(\bar{z}-\bar{z}_6)]^{1/3}}, \\ \bar{\partial}_{\bar{z}} \bar{X}_{cl}(\bar{z}) &= \frac{\tilde{a}(\bar{z}_4, \bar{z}_5, \bar{z}_6)}{[\bar{z}(\bar{z}-1)(\bar{z}-\bar{z}_4)(\bar{z}-\bar{z}_5)(\bar{z}-\bar{z}_6)]^{2/3}}, \\ \partial_z \bar{X}_{cl}(z) &= \frac{\tilde{b}(z_4, z_5, z_6)}{[z(z-1)(z-z_4)(z-z_5)(z-z_6)]^{1/3}}. \end{aligned} \quad (3.17)$$

Here,  $z_4, z_5, z_6$  are the three vertex insertion locations which cannot be fixed by  $SL(2, \mathbf{C})$  invariance while the first three vertices  $z_1, z_2, z_3$  have been sent



to 0, 1,  $\infty$  resp. The functions  $a, b, \tilde{a}, \tilde{b}$  depend on the unfixed vertex locations and are determined for each classical solution  $X_{cl}$  using *global* monodromy conditions. The classical action is given by

$$S_{cl} = \frac{1}{4\pi} \int d^2z (\partial X_{cl} \bar{\partial} \bar{X}_{cl} + \bar{\partial} X_{cl} \partial \bar{X}_{cl}). \quad (3.18)$$

Upon substitution of formulae (3.17) into (3.18), one finds it necessary to perform the integrals

$$I_r = \int \frac{d^2z}{|z(z-1)(z-z_4)(z-z_5)(z-z_6)|^r}, \quad r = \frac{4}{3}, \frac{2}{3}. \quad (3.19)$$

Using techniques developed in [32], one can show that both of these integrals may be written in the form of sums of products of integrals along the real axis. The expressions obtained are typically complicated special functions of the unfixed vertex locations. Rather than attempting the calculation, we merely write the results symbolically as<sup>8</sup>

$$I_r = \sum_i c_r^i F_i(r; z_4, z_5, z_6) \bar{G}_i(r; \bar{z}_4, \bar{z}_5, \bar{z}_6). \quad (3.20)$$

It follows that

$$S_{cl}(z_4, z_5, z_6; \bar{z}_4, \bar{z}_5, \bar{z}_6) = \frac{1}{4\pi} (a\tilde{a}I_{4/3} + b\tilde{b}I_{2/3})(z_4, z_5, z_6; \bar{z}_4, \bar{z}_5, \bar{z}_6). \quad (3.21)$$

The functions  $a, b, \tilde{a}, \tilde{b}$  are also complicated special functions of the unfixed vertex locations. It should be clear that  $S_{cl}$  is a horrendous function. What is more, this action must be exponentiated, summed over an infinity of classical solutions  $X_{cl}$ , and multiplied by the quantum part of the partition function to obtain the correlator:

$$\begin{aligned} & \langle V_1(0,0)V_2(1,1)V_3(\infty,\infty)V_4(z_4,\bar{z}_4)V_5(z_5,\bar{z}_5)V_6(z_6,\bar{z}_6) \rangle \\ &= Z(z_4, z_5, z_6; \bar{z}_4, \bar{z}_5, \bar{z}_6) \\ &= Z_{qu}(z_4, z_5, z_6; \bar{z}_4, \bar{z}_5, \bar{z}_6) \cdot \sum_{X_{cl}} \exp[-S_{cl}(z_4, z_5, z_6; \bar{z}_4, \bar{z}_5, \bar{z}_6)]. \end{aligned} \quad (3.22)$$

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<sup>8</sup> At the risk of annoying the reader, we have explicitly shown the dependence on unfixed vertex locations, in order that we might stress wherein the difficulty lies.

Here, the quantum part of the partition function  $Z_{qu}(z_4, z_5, z_6; \bar{z}_4, \bar{z}_5, \bar{z}_6)$  will be some other horrific function of the unfixed vertex locations. Finally, we must extract the effective field theory coupling coefficient by integrating the unfixed vertex locations over the complex plane:

$$f_{i_1 \dots i_6} \propto \int d^2 z_4 d^2 z_5 d^2 z_6 Z(z_4, z_5, z_6; \bar{z}_4, \bar{z}_5, \bar{z}_6). \quad (3.23)$$

Integrating over exponentials of sums of products of special functions of several complex variables is bad enough, but one also has the  $Z_{qu}$  prefactor and the functions  $a, b, \tilde{a}, \tilde{b}$  to deal with. Suffice it to say, the explicit calculation of coupling coefficients for higher dimensional twisted couplings certainly looks like a major undertaking.

As a result, we take a more phenomenological approach, using modular covariance as a guide. In this respect our effective field theory is “string-inspired” rather than “string-derived”. It is our hope that by appealing to symmetries of the underlying theory, we will capture the most important features of the *bona fide* couplings. Modular covariant  $3m$ -dimensional twisted couplings can be constructed by tensoring the polynomials (3.13). This leads us to the implicit definition of  $T$ -dependent twisted coupling coefficients  $f_{i_1 \dots i_{3m}}^T$  given by

$$\sum_{\{i_j\}} f_{i_1 \dots i_{3m}}^T \Phi_1^{i_1} \dots \Phi_{3m}^{i_{3m}} = \frac{1}{m!(3!)^m} \sum'_{\{n_j\}} \prod_{k=0}^{m-1} p(T; \Phi_{n_{3k+1}}, \Phi_{n_{3k+2}}, \Phi_{n_{3k+3}}), \quad (3.24)$$

Here,  $\sum'_{\{n_j\}}$  indicates that the  $3m$ -tuple of subscripts  $(n_1, \dots, n_{3m})$  should be summed over all permutations of  $(1, 2, \dots, 3m)$ . The factor  $1/m!(3!)^m$  accounts for trivial permutation symmetries. This construction treats the different species of fields  $\Phi_1, \dots, \Phi_{3m}$  in a symmetric way with respect to fixed point couplings. If we define the  $3m$ -dimensional twisted superpotential coupling as

$$\lambda f_{i_1 \dots i_{3m}}^T [\eta(T)]^{2(\sum q_n - 1)} \Phi_1^{i_1} \dots \Phi_{3m}^{i_{3m}}, \quad (3.25)$$

it can be checked that this will satisfy the requirement (3.8). We generically denote the overall modulus- and fixed-point-independent coupling strength by  $\lambda$ . The actual value of this strength will depend on the dimensionality of the coupling and the species of fields which enter. For  $m > 1$  in (3.25), it

is likely that  $\lambda \gg 1$ , due to the numerous world-sheet integrals which must be performed in (3.23) above, as pointed out recently by Cvetič, Everett and Wang [17]. This aspect of nonrenormalizable couplings in string-derived models has been overlooked in much of the earlier literature, due to the temptation to estimate unknown quantities as  $\mathcal{O}(1)$ .

All of the above considerations dealt with a two-dimensional orbifold. We must generalize our results to the six-dimensional case. Also, it is necessary to say what should be done if untwisted states appear in a coupling or if some of the twisted states have nonzero oscillator numbers. Here, we address these complications only to the extent that it is necessary for the quark mass couplings in the toy model which is to be discussed in Section 4. Again, we take a phenomenological approach, using modular covariance as a guide, rather than attempting to explicitly derive effective field theory coupling coefficients from the underlying conformal field theory. The reason once again is that the calculations appear to be extremely difficult.

For a six-dimensional  $Z_3$  orbifold the twist field operators generalize to  $\sigma_{ijk}(z, \bar{z})$ , where the triple  $ijk$  denotes which of the 27 fixed points the twisted field  $\Phi_n^{ijk}$  created by the vertex operator sits at. The indices refer to the fixed point locations in each of the three complex planes. To the extent that the three complex planes are orthogonal to each other, which is the case if the off-diagonal T-moduli vanish, it is possible to decompose the twist operators into a tensor product of two-dimensional orbifold twist operators:

$$\sigma_{ijk}(z, \bar{z}) \equiv \sigma_i^1(z, \bar{z}) \otimes \sigma_j^2(z, \bar{z}) \otimes \sigma_k^3(z, \bar{z}). \quad (3.26)$$

Then the six-dimensional orbifold couplings are a tensor product of the two-dimensional ones. Indeed, it can be checked that given the transformation properties of the two dimensional twist operators,<sup>9</sup>

$$\prod_{I=1}^3 p(T^I; \tilde{\sigma}^I(z_1, \bar{z}_1) \tilde{\sigma}^I(z_2, \bar{z}_2) \tilde{\sigma}^I(z_3, \bar{z}_3)) \rightarrow \prod_{I=1}^3 e^{i\phi(a^I, b^I, c^I, d^I)} p(T^I; \tilde{\sigma}^I(z_1, \bar{z}_1) \tilde{\sigma}^I(z_2, \bar{z}_2) \tilde{\sigma}^I(z_3, \bar{z}_3)). \quad (3.27)$$

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<sup>9</sup>Here,  $\tilde{\sigma}_i^I = e^{i\beta(T^I, \bar{T}^I)/2} \sigma_i^I$  generalizes the  $\tilde{\sigma}_i$  defined above for the two-dimensional orbifold case.

This has the correct transformation, up to the factor  $\prod_I \exp(-\sum_i q_i^I F(T^I))$  which would come from the transformation properties of the matter field vertex operators not accounted for by  $\tilde{\sigma}_i^I$ . Then the generalization of (3.4) with the correct modular transformation properties is:

$$\lambda \cdot \left( \prod_I [\eta(T^I)]^2 f_{i_1^I i_2^I i_3^I}^{T^I} \right) \Phi_1^{i_1^1 i_1^2 i_1^3} \Phi_2^{i_2^1 i_2^2 i_2^3} \Phi_3^{i_3^1 i_3^2 i_3^3} \quad (3.28)$$

It is easy to check that this holds for the higher dimensional couplings as well. We simply take the obvious products of 2-d orbifold coupling coefficients:

$$\lambda \cdot \left( \prod_I [\eta(T^I)]^{2(\sum_n q_n^I - 1)} f_{i_1^I \dots i_{3m}^I}^{T^I} \right) \Phi_1^{i_1^1 i_1^2 i_1^3} \dots \Phi_{3m}^{i_{3m}^1 i_{3m}^2 i_{3m}^3}. \quad (3.29)$$

Now we consider the occurrence of twisted oscillator fields and untwisted fields in higher dimensional couplings. The vertex operator for a twisted oscillator field is proportional to an excited twist operator. The excited twist operator can be written in terms of an ordinary twist operator and a factor of  $\bar{\partial} \tilde{X}^\ell$ , with  $\ell$  depending on the oscillator direction [27]. Then the classical action  $S_{cl}$  is still computed in the presence of the same twist operators and we expect that the fixed point dependence should be the same as in the case where none of the twisted states were oscillators. Indeed, this has been found to be the case in  $Z_N$  orbifolds where renormalizable couplings may involve oscillator fields [33]. It has been shown [30, 31] that the modular weight of an  $N_L = 1/3$  oscillator state  $Y^{\ell,ijk}$  (where  $\ell$  is the direction of the oscillator and  $ijk$  specifies the fixed point location) is given by

$$q^I(Y^{\ell,ijk}) = (2/3, 2/3, 2/3) + \delta_\ell^I. \quad (3.30)$$

Since the vertex operator creating the twisted oscillator state is proportional to  $\tilde{\sigma}_i^1 \otimes \tilde{\sigma}_j^2 \otimes \tilde{\sigma}_k^3$ , we expect that the state transforms under  $T^I \rightarrow 1/T^I$  according to (3.10) if  $I \neq \ell$ , while the power  $2/3$  should be replaced by  $5/3$  if  $I = \ell$ . Based on this assumption, the modular invariant couplings involving oscillator fields (which for the  $Z_3$  orbifold are always higher dimensional couplings because of the automorphism selection rule) are obtained from (3.29) directly, with the oscillator nature of states reflected in the modular weights  $q_i^I$  and a different overall strength  $\lambda$  than would be obtained if the

states were not oscillators, due to the presence of the additional operator  $\bar{\partial}\bar{X}^\ell$ . Obviously, adding untwisted states to a coupling does not introduce any new twist operators. We can always choose a “picture” such that the vertex operator of the untwisted state  $U^i$  goes like  $\partial X^i$ . Then the change in the conformal field theory correlator will be completely in the quantum part and we expect the fixed point dependence to be unchanged. An untwisted state  $U^i$  has modular weight  $q_i^I = \delta_i^I$ . As a result of these arguments, we conclude that the coefficients for a coupling with  $3m$  twisted fields and  $n$  untwisted fields can be read off from (3.29), only we must include the modular weights of the untwisted fields in the sums in the exponents of the  $\eta$  functions, and the overall coupling strength  $\lambda$  will be different than if no untwisted fields were in the coupling.

## 4 Toy model

Below the scale of  $U(1)_X$  breaking,  $\Lambda_X \sim 10^{17}$  GeV, the quark and higgs spectrum is assumed to be that of the Minimal Supersymmetric Standard Model (MSSM). Extra color triplets get vector mass couplings when Xiggses get vevs. As a consequence, they get masses  $\mathcal{O}(\Lambda_X)$  and integrate out of the spectrum near the string scale. Above  $\Lambda_X$  we suppose the spectrum contains untwisted doublet quarks  $Q^i$ , untwisted  $u_L^c$ -like quarks  $u_1^i$ , twisted  $u_L^c$ -like quarks  $u_2^i$ , twisted  $d_L^c$ -like quarks  $d^i$ , untwisted  $H_u$ -like higgs doublets  $H_u^i$  and twisted  $H_d$ -like higgs doublets  $H_d^i$ . These assignments are quite similar to those of the FIQS model. The superscript on untwisted fields corresponds to the H-momentum of the states in the underlying theory, and takes values  $i = 1, 2, 3$ . The superscript on twisted fields corresponds to the fixed point location in the third complex plane of the six-dimensional compact space. As in the FIQS model, three linear combinations of the six  $u_L^c$ -like quarks survive in the low-energy spectrum, which we describe by mixing matrices  $X^1$  and  $X^2$ :

$$u_1^i = X_{ij}^1 u_{jL}^c + \text{heavy}, \quad u_2^i = X_{ij}^2 u_{jL}^c + \text{heavy}. \quad (4.1)$$

The mixings to  $\Lambda_X$  scale mass eigenstates, denoted by “heavy”, are not important to our tree level analysis of low energy quark mass matrices. We

assume that all extra higgses integrate out near the scale  $\Lambda_X$  due to vector couplings induced by the Xiggs vevs (as in the FIQS model), leaving one pair which we identify as the  $H_u$  and  $H_d$  of the MSSM:

$$H_u = H_u^1, \quad H_d = H_d^3. \quad (4.2)$$

We introduce SM singlet Xiggses  $Y_1^{\ell i}, Y_2^{\ell i}$  and  $Y_3^{\ell i}$  which get  $\mathcal{O}(\Lambda_X)$  vevs and appear in the nonrenormalizable mass couplings of the quarks. The  $Y$ 's are charged under  $U(1)_X$  and their scalar components are among those which get vevs to cancel the  $U(1)_X$  FI term. The  $Y$ 's are twisted states with nonzero left-moving oscillator number  $N_L = 1/3$ . The first superscript corresponds to the oscillator direction in the compact space,  $\ell = 1, 2, 3$ . The second superscript corresponds to the fixed point location in the third complex plane of the compact space. Such fields also arise in the FIQS model. In what follows, we use the same symbols for superfields and scalar components of fields other than the quarks, with the meaning obvious by context. Similarly, whether we refer to a quark superfield or its fermionic component should be obvious by context.

We assume that the leading holomorphic couplings giving quarks masses are contained in the superpotential

$$\begin{aligned} W_{qm} = & \lambda_0 |\epsilon_{ijk}| H_u^i Q^j u_1^k + \lambda_1 \lambda_{i_1 i_2 i_3 \ell_1 \ell_2 k j} Y_1^{\ell_1 i_1} Y_1^{\ell_2 i_2} H_u^k Q^j u_2^{i_3} \\ & + \lambda_2 \lambda_{i_1 i_2 i_3 i_4 i_5 i_6 \ell_1 \ell_2 \ell_3 \ell_4 j} Y_1^{\ell_1 i_1} Y_2^{\ell_2 i_2} Y_3^{\ell_3 i_3} Y_3^{\ell_4 i_4} H_d^{i_5} Q^j d^{i_6}. \end{aligned} \quad (4.3)$$

The trilinear untwisted coupling is proportional to  $|\epsilon_{ijk}|$  according to the *conservation of H-momentum* orbifold selection rule. The fields  $Y_{1,2,3}^{\ell i}$  are assumed to be charged identically under  $U(1)_X$ . We assume that the operators contained in (4.3) are each neutral under the full rank sixteen gauge group  $G$  obtained from the orbifold embedding. Thus, the other fields are assumed to have  $U(1)_X$  charges<sup>10</sup> such that each coupling is  $U(1)_X$  neutral. We assume that off-diagonal T-moduli  $T^{I\bar{J}}$ ,  $I \neq \bar{J}$ , have vanishing vevs, as in the FIQS model, so that the leading order kinetic terms for the matter fields are diagonal. In the FIQS model, nonvanishing off-diagonal T-moduli lead

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<sup>10</sup> In the three generation constructions presently under consideration, the  $U(1)_X$  charges are independent of generation number. Thus, horizontal flavor symmetries, such as those considered in refs. [20, 34], cannot be implemented in this context.

to nonvanishing F-terms which break supersymmetry at the scale  $\Lambda_X$ , which is unacceptable. Though these fields may acquire vevs once supersymmetry is broken in the hidden sector, we expect the vevs to be at most of order the hidden sector supersymmetry breaking scale. As a result, off-diagonal T-moduli give negligible contributions to the kinetic terms of the quarks. We further assume that the diagonal T-moduli  $T^I \equiv T^{I\bar{I}}$  are stabilized at one of their self-dual points  $\langle T^I \rangle = 1, e^{i\pi/6}$  once supersymmetry is broken, consistent with models of hidden sector supersymmetry breaking by gaugino condensation [12, 6]. It has also been argued that the T-moduli may stabilize at other points on the unit circle [6]. Either way, it would appear that string-derived scalar potentials for the T-moduli stabilize them to values  $|T^I| = 1$ . For this reason we view models which allow  $\langle T^I \rangle$  as large as required to obtain hierarchies in the Yukawa couplings of twisted fields [35] to be unmotivated.

The Kähler metric for matter fields in (0,2)  $Z_3$  orbifolds (arbitrary Wilson lines and point group embeddings) has been determined to leading order [31, 36]. In the case of vanishing off-diagonal T-moduli, the metric of the untwisted fields  $Q^i$  and  $u_1^i$  is given by

$$\langle K_Q \rangle_{i\bar{\ell}} = \langle K_{u_1} \rangle_{i\bar{\ell}} = \delta_{i\bar{\ell}} \langle T^i + \bar{T}^{\bar{i}} \rangle^{-1}. \quad (4.4)$$

We make redefinitions  $Q^i \rightarrow \langle T^i + \bar{T}^{\bar{i}} \rangle^{1/2} Q^i$  and similarly for  $u_1^i$ . Similar arguments hold for the twisted fields  $u_2^i$  and  $d^i$ , whose Kähler metric at leading order is

$$\langle K_d \rangle_{i\bar{\ell}} = \langle K_{u_2} \rangle_{i\bar{\ell}} = \delta_{i\bar{\ell}} \prod_I \langle T^I + \bar{T}^{\bar{I}} \rangle^{-2/3}. \quad (4.5)$$

We assume that the overall coupling strengths  $\lambda_0, \lambda_1, \lambda_2$  in (4.3) reflect these rescalings and that the quark fields entering these couplings are the rescaled ones. We also assume that the factor  $\exp \langle K \rangle / 2$  has been absorbed into these coupling strengths and make use of the fact that  $\langle \mathcal{D}_i D_j W \rangle \approx \langle W_{ij} \rangle$  is a very good approximation, in the supergravity lagrangian notation of Wess and Bagger [37]. It can be checked that the terms which we drop are  $\mathcal{O}(m_{\tilde{G}} m_W / m_P)$ , where  $m_{\tilde{G}}$  is the gravitino mass. When working with these rescaled fields, we may raise and lower indices with impunity since their

Kähler metric is canonical in the leading order approximation made here. Once the fields  $u_1^i$  and  $u_2^i$  have been rescaled in this way, the mixings to mass eigenstates  $u_{iL}^c$  and their three heavy relatives (all canonically normalized) can be made unitary. We then have as a constraint:

$$(X^{1\dagger}X^1)_{jk} + (X^{2\dagger}X^2)_{jk} = \delta_{jk}. \quad (4.6)$$

When one takes (4.1) and (4.2) into account, the effective Yukawa couplings for the quarks are given by

$$\begin{aligned} \lambda_{jm}^u &= \lambda_0(\delta_{j2}X_{3m}^1 + \delta_{j3}X_{2m}^1) + \lambda_1\lambda_{i_1i_2i_3\ell_1\ell_21j}\langle Y_1^{\ell_1i_1}Y_1^{\ell_2i_2}\rangle X_{i_3m}^2, \\ \lambda_{jm}^d &= \lambda_2\lambda_{i_1i_2i_3i_43m\ell_1\ell_2\ell_3\ell_4j}\langle Y_1^{\ell_1i_1}Y_2^{\ell_2i_2}Y_3^{\ell_3i_3}Y_3^{\ell_4i_4}\rangle. \end{aligned} \quad (4.7)$$

In going from (4.3) to (4.7), we have set  $k = 1$  in the second coupling of (4.3) because  $H_u = H_u^1$  and we have fixed  $i_5 = 3$  and  $i_6 = m$  in the third coupling of (4.3) since we couple to  $H_d = H_d^3$  and  $d^m$ .

We now apply the assumptions of Section 3 to the toy model. The toy model is based on string-derived models where two “discrete” Wilson lines are included in the embedding to give a three generation model [14, 13]. By construction, states which differ only by their fixed point location in the third complex plane have identical gauge quantum numbers. On the other hand, states which differ by fixed point locations in the first two complex planes generally have different gauge quantum numbers under the rank 16 gauge group which survives the orbifold compactification. Typically, the embedding is arranged so that the rank 16 gauge group has the form  $SU(3) \times SU(2) \times [U(1)]^m \times G_c$ , where  $G_c$  is a simple group which condenses in the hidden sector to break supersymmetry. The extra  $U(1)$ ’s get broken down to  $U(1)_Y \times$  (hidden  $U(1)$ ’s) by the FI term associated with the anomalous  $U(1)_X$ . Fixed point locations in the first two complex planes become species labels. In what follows, the only fixed point location superscript on twisted states is that corresponding to the third complex plane. This serves as a family number for twisted states in these models. Twisted “relatives” differ only by their fixed point location in the third complex plane, so in many respects the effective Yukawas behave as if we were working with a two-



dimensional orbifold. The coupling coefficients in (4.7) are given by:

$$\begin{aligned}\lambda_{i_1 i_2 i_3 \ell_1 \ell_2 1 j} &= \left( \prod_I \eta(T^I)^{2(Q_1^I - 1)} \right) \chi_0(T^1) \chi_0(T^2) f_{i_1 i_2 i_3}^{T^3} \\ &\quad \text{if } (\ell_1, \ell_2) = (\underline{1}, j), \\ &= 0 \quad \text{else;}\end{aligned}\tag{4.8}$$

$$\begin{aligned}\lambda_{i_1 i_2 i_3 i_4 3 m \ell_1 \ell_2 \ell_3 \ell_4 j} &= \left( \prod_I \eta(T^I)^{2(Q_2^I - 1)} \right) \chi_1(T^1)^2 \chi_1(T^2)^2 f_{i_1 i_2 i_3 i_4 3 m}^{T^3} \\ &\quad \text{if } (\ell_1, \ell_2, \ell_3, \ell_4) = (\underline{1}, \underline{2}, \underline{3}, j), \\ &= 0 \quad \text{else;}\end{aligned}\tag{4.9}$$

$$Q_1^I = 2 + 2\delta_1^I + 2\delta_j^I, \quad Q_2^I = 5 + 2\delta_j^I.\tag{4.10}$$

The constraints on the allowed values of  $\ell_i$  follow from the automorphism selection rule; underlining denotes that any permutation of entries is permitted. The coefficients  $f_{i_1 i_2 i_3}^T$  carry the dependence on third complex plane fixed point locations of twisted fields appearing in the nonrenormalizable up-type quark mass coupling, and are given explicitly in (3.5) above. The third complex plane fixed point dependence for the down-type quark mass coupling follows from the six-dimensional twisted coupling, and is defined implicitly by (3.24). The six twist coupling coefficients  $f_{i_1 i_2 i_3 i_4 i_5 i_6}^T$  vanish by the *lattice group selection rule* unless  $i_1 + \dots + i_6 = 0 \bmod 3$ . It can be checked that the choices  $i_1, \dots, i_6$  satisfying this rule can be divided into four classes, depending on whether triples of the indices can be formed where the entries of the triples are either all the same (*s*) or all different (*d*). Members of the same class have identical values for  $f_{i_1 i_2 i_3 i_4 i_5 i_6}^T$ . The nonvanishing values of  $f_{i_1 i_2 i_3 i_4 i_5 i_6}^T$  are given in Table 2, according to which of the four classes the indices belong to. A representative example  $(i_1 \dots i_6)$  for each class is given to avoid any confusion. The factor of  $\chi_1(T^1)^2 \chi_1(T^2)^2$  in (4.9) follows from an additional assumption of our model: the fixed point locations (of the six species of twisted fields in the down-type Yukawa coupling) in the first two complex planes are such that the lattice group selection rule in each of the two planes is satisfied in the (*dd*) way of Table 2.

The strengths  $\lambda_0, \lambda_1, \lambda_2$ , the mixing matrices  $X_{ij}^1, X_{ij}^2$ , and the background  $\langle Y_{1,2,3}^{\ell_i} \rangle$  are treated as phenomenological parameters. We tune the

class	$f_{i_1 \dots i_6}^T$	representative	class	$f_{i_1 \dots i_6}^T$	representative
$ss$	$10\chi_0(T)^2$	(111111)	$sd$	$2\chi_0(T)\chi_1(T)$	(111123)
$ss'$	$\chi_0(T)^2$	(111222)	$dd$	$\chi_1(T)^2$	(123123)

Table 2: Six twist coupling coefficients by class

couplings, mixings and vevs to values which yield a reasonable phenomenology. In principle, all of these quantities would be fixed by a full and complete analysis of the string-derived effective supergravity. However, some of the background fields in  $\langle Y_{1,2,3}^{\ell_i} \rangle$  are D-moduli [21]; in order to fix these we must say how the D-moduli flat directions are lifted. In the reference just cited it was suggested how these flat directions may be lifted by nonperturbative effects in the hidden sector, via superpotential couplings of the D-moduli to hidden sector matter condensates. It should also be noted that the symmetries which give rise to the D-moduli are only valid for the classical scalar potential under the assumption of vanishing of F-terms for the D-moduli in the background. As a result, they are pseudo-Goldstone bosons and we expect that the D-moduli flat directions will also be lifted by loop corrections.<sup>11</sup> Preliminary estimates show that the D-moduli get contributions to their masses of order the gravitino mass from either effect. We are currently exploring how the phases of the Xiggses may be fixed by these mechanisms and what effect this will have on the KM phase in models of the type discussed here. Our results will be presented elsewhere.

We do not intend to be exhaustive in our analysis of the phenomenology of (4.7). Rather, we would simply like to demonstrate that it is possible to obtain a quark phenomenology which is consistent with experimental data. The shortest route to this goal is to implement textures in the effective Yukawa couplings. In this way our scan over parameter space is biased toward viable models. We make use of the results of a recent analysis of viable mass matrices [38], though we will not impose the hermiticity constraint implemented

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<sup>11</sup>We thank Korkut Bardakci for bringing this to our attention.

there since we have no motivation for it in the present context. We impose the textures

$$\lambda^u = f_t \begin{pmatrix} A_u \theta^8 & 0 & C_u \theta^4 \\ 0 & D_u \theta^4 & E_u \theta^2 \\ \tilde{C}_u \theta^4 & \tilde{E}_u \theta^2 & 1 \end{pmatrix}, \quad \lambda^d = f_b \begin{pmatrix} 0 & B_d \theta^3 & 0 \\ \tilde{B}_d \theta^3 & D_d \theta^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4.11)$$

through an arrangement of the mixing matrices  $X_{ij}^1, X_{ij}^2$  and vevs  $\langle Y_{1,2,3}^{\ell i} \rangle$ . Here,

$$\theta \approx V_{us} \approx 0.22, \quad f_t \approx m_t / \langle H_u^0 \rangle, \quad f_b \approx m_b / \langle H_d^0 \rangle, \quad (4.12)$$

$$A_u, C_u, \tilde{C}_u, D_u, E_u, \tilde{E}_u, B_d, \tilde{B}_d, D_d \sim \mathcal{O}(1). \quad (4.13)$$

The forms (4.11) were obtained by imposing the following textures in  $X^{1,2}$  and  $Y_{1,2,3}$ :

$$X^1 = \begin{pmatrix} * & * & * \\ r_1 \theta^4 & r_2 \theta^2 & r_3 \\ 0 & r_4 \theta^4 & r_5 \theta^2 \end{pmatrix}, \quad X^2 = \begin{pmatrix} * & * & * \\ * & * & * \\ s_1 \theta^4 & 0 & s_2 \end{pmatrix}, \quad (4.14)$$

$$\langle Y_1^{\ell i} \rangle = y_1 \theta^2 \delta_1^\ell \delta_3^i, \quad \langle Y_2^{\ell i} \rangle = y_2 \delta_2^\ell \delta_3^i, \quad \langle Y_3 \rangle = \begin{pmatrix} y_{31} \theta^3 & 0 & 0 \\ y_{32} \theta^2 & \tilde{y}_{31} \theta^3 & 0 \\ 0 & 0 & y_{33} \end{pmatrix}, \quad (4.15)$$

The elements denoted by  $*$  in (4.14) are left unspecified since they do not appear in the effective Yukawa matrices. We assume that they are chosen such that (4.6) is satisfied, which is generally true provided  $|r_3|^2 + |s_2|^2 + |r_5|^2 \theta^4 \leq 1$  because of the  $\theta^n$  suppressions on entries of the other columns. Up to this restriction, the quantities  $r_i, s_i, y_i, y_{ij}, \tilde{y}_{31} \sim \mathcal{O}(1)$ . We note that there is no inconsistency in having Xiggs vevs larger than the FI term, since the vevs of fields having opposite  $U(1)_X$  charge can be played off against each other in the  $U(1)_X$  D-term. As an example, in the FIQS model the Y-type Xiggses can be made arbitrarily large while maintaining D-flatness by simultaneously increasing some of the vevs of non-oscillator Xiggses which they denote by  $S_6^i$ . Of course at some point the nonlinear  $\sigma$ -model perturbation theory breaks down.

Given the assumptions enumerated above, the effective Yukawa matrices take the form ( $T_c^I \equiv \langle T^I \rangle$ ):

$$\lambda^u = \lambda_0 \begin{pmatrix} h_u s_1 \theta^8 & 0 & h_u s_2 \theta^4 \\ 0 & r_4 \theta^4 & r_5 \theta^2 \\ r_1 \theta^4 & r_2 \theta^2 & r_3 \end{pmatrix}, \quad (4.16)$$

$$h_u \equiv \frac{\lambda_1}{\lambda_0} \eta(T_c^1)^{10} \eta(T_c^2)^2 \eta(T_c^3)^2 \chi_0(T_c^1) \chi_0(T_c^2) \chi_0(T_c^3) y_1^2, \quad (4.17)$$

$$\lambda^d = h_d \begin{pmatrix} 0 & 2\eta(T_c^1)^4 \chi_1(T_c^3) y_{31} \theta^3 & 0 \\ 2\eta(T_c^2)^4 \chi_1(T_c^3) \tilde{y}_{31} \theta^3 & 2\eta(T_c^2)^4 \chi_1(T_c^3) y_{32} \theta^2 & 0 \\ 0 & 0 & 5\eta(T_c^3)^4 \chi_0(T_c^3) y_{33} \end{pmatrix}, \quad (4.18)$$

$$h_d \equiv 2\lambda_2 \left[ \eta(T_c^1) \eta(T_c^2) \eta(T_c^3) \right]^8 \chi_1(T_c^1)^2 \chi_0(T_c^2)^2 \chi_0(T_c^3) y_1 y_2 y_{33} \theta^2. \quad (4.19)$$

The quantities  $B_d, \tilde{B}_d, D_d$  in (4.11) can be varied independently by adjusting the ratios  $y_{31}/y_{33}, \tilde{y}_{31}/y_{33}, y_{32}/y_{33}$ . The ratio of heavy generation Yukawa eigenvalues  $f_b/f_t$  can be varied independently of  $B_d, \tilde{B}_d, D_d$  and  $\lambda^d$  by adjusting  $y_2 y_{33}^2/y_1$ . The top quark Yukawa eigenvalue  $f_t$  can be adjusted independently by varying  $r_3$ . However, if  $\lambda_0$  is too small there may be a minimum  $\tan \beta$  below which we cannot match experimental data, since  $|r_3| < 1$  is required by (4.6). Recall that we have absorbed a factor  $\exp \langle K \rangle / 2$  into  $\lambda_0$ , as well as the effects of quark field rescalings to account for noncanonical kinetic terms. Typically,  $\exp \langle K \rangle / 2 < 1$  when the string moduli get  $\mathcal{O}(1)$  vevs, so this may be a worry. Without an explicit model of the superpotential couplings and Xiggs vevs which determine the mixing matrices  $X^{1,2}$ , it is not possible to say whether or not the entries of  $\lambda_u$  can be varied independently of each other and  $\lambda_d$ ; we will assume that this is true.

With the above assumptions, scanning over the Xiggs vevs and the mixing coefficients  $r_i, s_i$  for viable models is equivalent to varying the coefficients in (4.13) independently and tuning the values of  $f_t, f_b$  to agree with experimental data. We then rephase the quarks according to the convention

$$V_{ud} > 0, \quad V_{us} > 0, \quad V_{cb} > 0, \quad V_{ts} < 0, \quad V_{cd} < 0, \quad (4.20)$$

to which the Wolfenstein parameterization [39, 40] is an approximation. As is well known, the advantage of such a parameterization is that the elements with significant complex phase are the smallest ones,  $V_{ub}$  and  $V_{td}$ .

All of these calculations are done at the  $U(1)_X$  breaking scale, and are therefore subject to evolution under the renormalization group. The evolution of the quark masses and mixing angles assuming the MSSM spectrum has been studied extensively; approximate analytic formulas are available, for example in refs. [41, 18]. We will use the approximations of [18] to evolve the low energy data to the scale of  $U(1)_X$  breaking, which we assume to be  $\Lambda_X \sim \Lambda_s \sim 5 \times 10^{17}$  GeV, based on what occurs in the FIQS model.<sup>12</sup> The following quantities are approximately scale-independent:

$$\frac{m_c}{m_u}, \quad \frac{m_s}{m_d}, \quad |V_{ud}|, \quad |V_{cs}|, \quad |V_{tb}|, \quad |V_{us}|, \quad |V_{cd}|. \quad (4.21)$$

The running of the other quantities is approximately given by:

$$\left. \frac{m_t}{m_c} \right|_{\Lambda_X} = \frac{1}{\xi_t^3 \xi_b} \left. \frac{m_t}{m_c} \right|_{M_Z}, \quad \left. \frac{m_b}{m_s} \right|_{\Lambda_X} = \frac{1}{\xi_t \xi_b^3} \left. \frac{m_b}{m_s} \right|_{M_Z}, \quad |V_{cb}|_{\Lambda_X} = \xi_t \xi_b |V_{cb}|_{M_Z}; \quad (4.22)$$

the quantities  $|V_{ub}|, |V_{td}|, |V_{ts}|$  scale in the same manner as  $|V_{cb}|$ . The scaling functions are given by

$$\xi_{t,b} = \exp \left[ \frac{-1}{16\pi^2} \int_0^{\ln(\Lambda_X/M_Z)} d\chi \, f_{t,b}^2(\chi) \right], \quad (4.23)$$

where  $\chi = \ln(\mu/M_Z)$  and  $f_{t,b}(\mu)$  are the Yukawa coupling eigenvalues of the top and bottom quarks appearing in (4.11), at the scale  $\mu$ . We assume the scale of observable sector supersymmetry breaking is 1 TeV and we set  $\tan\beta = 5$ . For low energy data we use the values of the running quark masses at the scale  $M_Z$  as determined in ref. [42] and the CKM data listed in ref. [43]. Taking into account the errors quoted in these two source, we find the following values:

$$\begin{aligned} f_t(\Lambda_X) &= 0.74_{-0.24}^{+1.65}, & f_b(\Lambda_X) &= 0.028(4), \\ \left. \frac{m_t}{m_c} \right|_{\Lambda_X} &= 440_{-100}^{+390}, & \left. \frac{m_c}{m_u} \right|_{\Lambda_X} &= 290(60), \\ \left. \frac{m_b}{m_s} \right|_{\Lambda_X} &= 38(7), & \left. \frac{m_s}{m_d} \right|_{\Lambda_X} &= 20(4), \end{aligned}$$

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<sup>12</sup>See Appendix A.

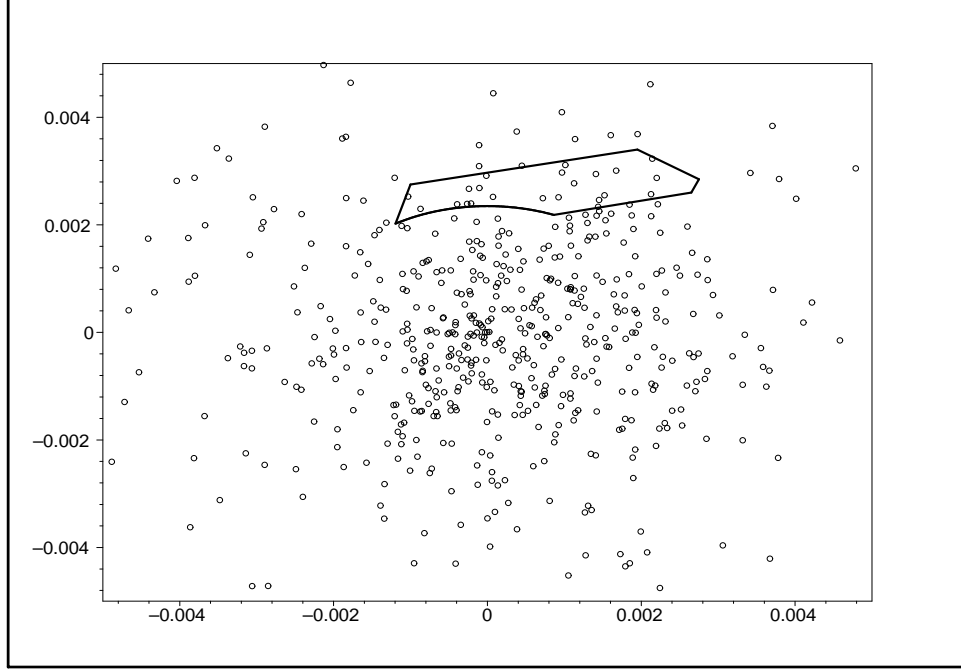


Figure 1:  $V_{ub}(M_Z)$  for complex mixing matrices  $X^{1,2}$  and Xiggs vevs  $\langle Y_{1,2,3}^{\ell i} \rangle$ . The T-moduli are stabilized at  $T_c^I = 1$ . For comparison, the experimentally preferred region [43] is outlined.

$$|V_{CKM}|_{\Lambda_X} = \begin{pmatrix} 0.9752(8) & 0.220(4) & 0.0027(12) \\ 0.220(4) & 0.9745(8) & 0.033(4) \\ 0.014(11) & 0.065(36) & 0.9992(2) \end{pmatrix}. \quad (4.24)$$

We stress that theoretical errors due to the approximations made in (4.21) and (4.22) have not been included in the estimates of uncertainty. However, for our purposes this is not an important issue since we can always make a small shift in the  $\mathcal{O}(1)$  parameters of our toy model to account for small corrections and larger uncertainties will just mean that more points in parameter space will give viable models.

In our analysis we consider both generic and extreme possibilities in order to get a feel for how the KM phase depends on the various sources of phases in (4.16) and (4.18).

*Case 1: generic mixings and Xiggs vevs*

We have scanned over the magnitudes of the parameters in (4.13) with Gaussian distributions centered on values suggested by the central values in (4.24) and with spreads suggested by the estimated uncertainties. Phases have been scanned on a flat distribution over the interval  $(-\pi, \pi]$ . We then compared mass ratios and the magnitudes of CKM elements, except  $|V_{ub}|$ , to the values in (4.24); if these results agreed with the values in (4.24), except  $|V_{ub}|$ , up to the stated uncertainties, we stored the values of  $V_{ub}(\Lambda_X)$ . We then scaled the magnitude of  $V_{ub}$  according to (4.22) but left the phase unrotated to get an estimate of  $V_{ub}(M_Z)$ . In Figure 1 we plot our results, showing only points near the acceptable region. The results are hardly surprising: if we allow the phases of the fields Xiggses to float randomly, the KM phase can take on any value we like. No magical cancellation occurs. Although regions of parameter space in this toy model do exist which have reasonable quark masses, mixings and CP violation, the model provides no understanding of why we live in one region of parameter space rather than another. All that can be said is that our model, which contains many more free parameters than the number of experimental data points which we are attempting to fit, can be made to agree with what is known about the quark sector. One promising point does emerge, however. Figure 1 shows that CP violation is generic in the toy model under consideration. To be fair, one could argue that we have gone through a lot of unnecessary work to prove the obvious: if nonrenormalizable couplings contribute significantly to the effective quark Yukawa matrices, and the Xiggses in these nonrenormalizable couplings get complex vevs, then CP violation is to be expected. However, as we discussed in Section 1, one can wonder whether the symmetry constraints of modular invariance and orbifold selection rules might render these phases spurious. We have explicitly shown that this is not the case.

#### *Case 2: complex Xiggs vevs*

Here, we make the quantities  $r_i, s_i$  in (4.14) real and positive and keep  $T_c^I = 1$  in order to isolate the effects of the phases of the Xiggs. With these assumptions it can be seen from (4.16) and (4.18) that the  $\mathcal{O}(1)$  coefficients (4.13) satisfy

$$\arg D_u = \arg E_u = \arg \tilde{C}_u = \arg \tilde{E}_u = 0, \quad (4.25)$$

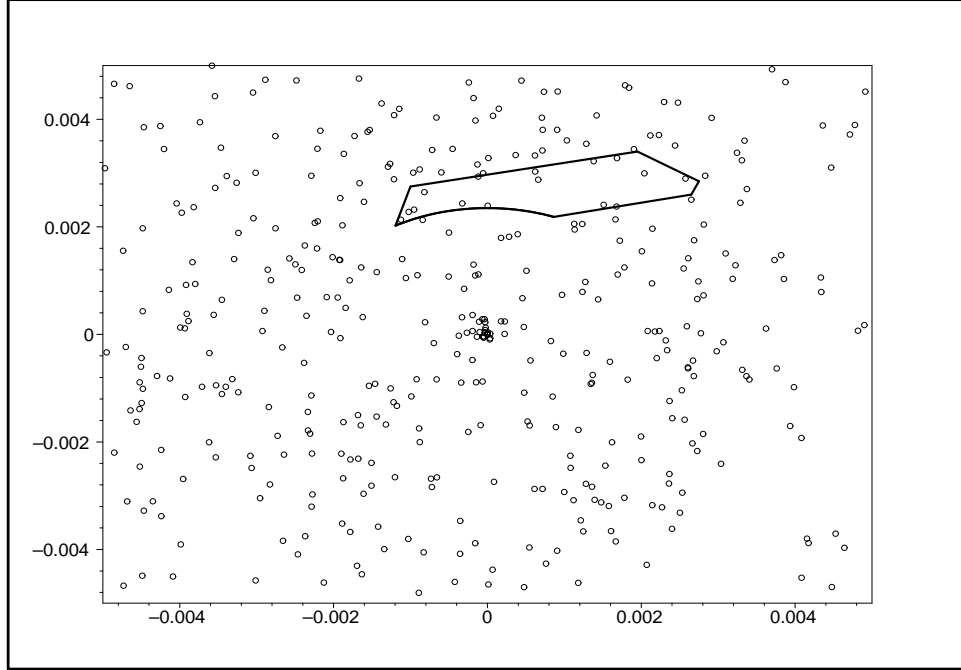


Figure 2:  $V_{ub}(M_Z)$  for real mixing matrices  $X^{1,2}$  and complex Xiggs vevs  $\langle Y_{1,2,3}^{\ell i} \rangle$ . The T-moduli are stabilized at  $T_c^I = 1$ .

$$\arg A_u = \arg C_u, \quad (4.26)$$

with  $\arg C_u$ ,  $\arg B_d$ ,  $\arg \tilde{B}_d$  and  $\arg D_d$  independent parameters to be scanned over. As in the previous case, we scan over the  $\mathcal{O}(1)$  magnitudes of the coefficients (4.13) using a Gaussian distribution, and plot values of  $V_{ub}(M_Z)$  for models which satisfy all constraints in (4.24) except the one on  $|V_{ub}|_{\Lambda_X}$ . The results are given in Figure 2. Comparing to Figure 1, it can be seen that whether or not the mixings  $X^{1,2}$  are a source of phase makes little difference. Complex Xiggs vevs provide a source of a KM phase and allow us to obtain any value we like.



*Case 3: complex T-moduli*

As discussed above, some of the T-moduli may stabilize at  $e^{i\pi/6}$ , their other self-dual point under  $SL(2, \mathbf{Z})$ . If we keep the mixing matrices  $X^{1,2}$  and Xiggs vevs  $\langle Y_{1,2,3}^{\ell i} \rangle$  complex, then the results are indistinguishable from those of Fig. 1. To isolate the effect of T-moduli sitting at the other self-dual point, we have constrained the coefficients  $r_i, s_i$  in (4.14) and the Xiggs vevs  $\langle Y_{1,2,3}^{\ell i} \rangle$  to be real and positive in what follows. Next, referring to (4.11), (4.16) and (4.18), we define

$$\gamma_1 \equiv \arg h_u = \arg A_u = \arg C_u, \quad (4.27)$$

$$\gamma_2 \equiv \arg B_d = \arg \frac{\eta(T_c^1)^4 \chi_1(T_c^3)}{\eta(T_c^3)^4 \chi_0(T_c^3)}, \quad (4.28)$$

$$\gamma_3 \equiv \arg \tilde{B}_d = \arg D_d = \arg \frac{\eta(T_c^2)^4 \chi_1(T_c^3)}{\eta(T_c^3)^4 \chi_0(T_c^3)}, \quad (4.29)$$

$$\Gamma \equiv \gamma_1 - \gamma_2 + \gamma_3. \quad (4.30)$$

It can be checked that the Yukawa matrices (4.16) and (4.18) can be rephased such that  $\lambda^d$  has all positive entries and

$$\arg \lambda^u = \begin{pmatrix} \Gamma & 0 & \Gamma \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (4.31)$$

This is easily implemented in a scan of the parameters in (4.13) by requiring all of them to be positive except  $\arg A_u = \arg C_u = \Gamma$ . Using Table 1 it is straightforward to determine  $\Gamma$ . We summarize the possible values in Table 3. The results of the scan are presented in Figure 3. Once again, these are values of  $V_{ub}(M_Z)$  for models which satisfy all constraints in (4.24) except the one on  $|V_{ub}|_{\Lambda_X}$ .

It can be seen that neither possibility is consistent with the experimentally preferred region. This result only rules out T-moduli as the sole source of CP violating phases in the toy model considered here. In another model the powers of  $\eta(T)$ ,  $\chi_0(T)$ ,  $\chi_1(T)$  would likely enter differently, since these depend on the dimension of a given nonrenormalizable coupling. It should also be noted that whereas we have set the mixing matrices  $X^{1,2}$  real, they typically have a nontrivial dependence on  $\arg T_c^I$ , since they are determined at least

$\arg T_c^1$	$\arg T_c^2$	$\arg T_c^3$	$\Gamma$	$\arg T_c^1$	$\arg T_c^2$	$\arg T_c^3$	$\Gamma$
0	0	0	0	0	$\pi/6$	$\pi/6$	$-\pi/6$
$\pi/6$	0	0	$-\pi/6$	$\pi/6$	0	$\pi/6$	$-\pi/6$
0	$\pi/6$	0	$-\pi/6$	$\pi/6$	$\pi/6$	0	$-\pi/3$
0	0	$\pi/6$	0	$\pi/6$	$\pi/6$	$\pi/6$	$-\pi/3$

Table 3: Phases from complex  $T_c^I$ .

in part by couplings involving twisted fields. Thus, the results of Figure 3 would likely change if this part of the model were made explicit.

## 5 Conclusions

In this article we have discussed several possible sources of CP violation in semi-realistic heterotic orbifold models. We have presented examples where CP violation does not occur in spite of the presence of phases, derived from complex string moduli vevs, in renormalizable coupling coefficients. However, it was described how nonrenormalizable couplings give a significant contribution to the effective quark Yukawa matrices when an anomalous  $U(1)_X$  is present and we argued that this generically leads to a nontrivial KM phase.

In order to make a detailed analysis of models with nonrenormalizable couplings, we introduced modular covariant nonrenormalizable superpotential couplings. It was explained why it is difficult to obtain the *bona fide* effective coupling coefficients from conformal field theory techniques. It was also pointed out that higher order terms in the Kähler potential should be important in cases where an anomalous  $U(1)_X$  is present. These theoretical uncertainties represent a significant stumbling block to further progress in string-derived effective supergravity models and it is hoped that they will be resolved at some point in the near future.

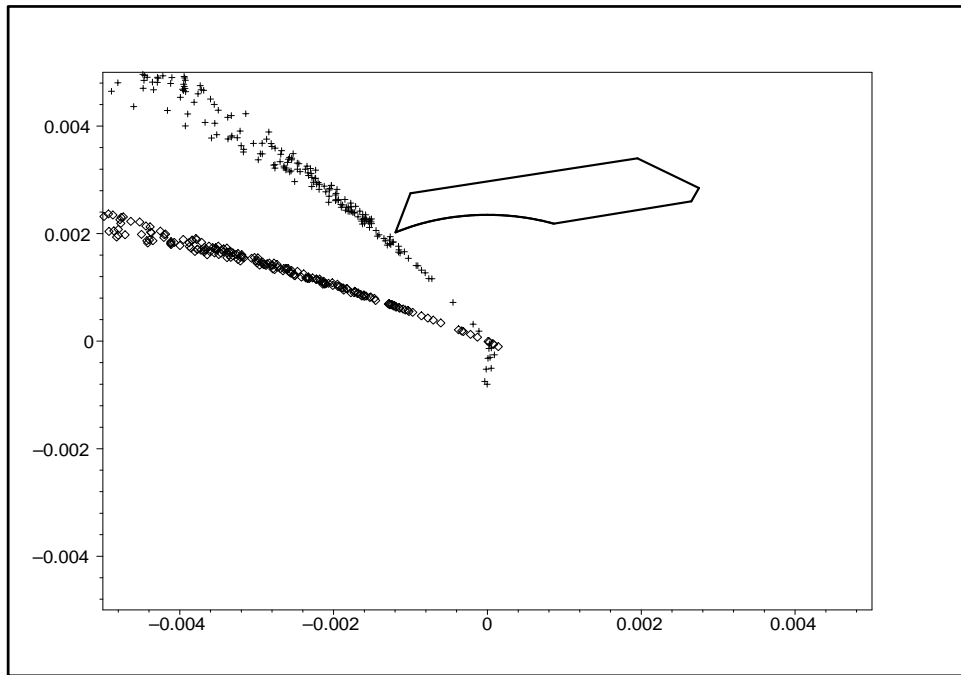


Figure 3:  $V_{ub}(M_Z)$  for positive parameters except  $\Gamma$ . The two nontrivial possibilities are displayed:  $\Gamma = -\pi/3$  (crosses) and  $\Gamma = -\pi/6$  (diamonds).

The KM phase was determined explicitly in a toy model inspired by three-generation heterotic  $Z_3$  orbifold constructions. Though target space modular invariance and orbifold selection rules greatly restrict the coupling coefficients of nonrenormalizable couplings, we found it possible to obtain viable Yukawa couplings for quarks by adjusting the vevs of Xiggses. This result highlights the necessity of understanding how D-moduli flat directions are lifted in a given model. In principle the Xiggs vevs should be determined by the mechanisms which lift these flat directions. This would eliminate our ability to tune the scalar background to our liking and would in most cases probably render the quark phenomenology inconsistent with low energy data. We are currently investigating this issue and hope to report on it in a future publication.

## Acknowledgements

The author would like to thank Prof. Mary K. Gaillard for innumerable discussions and helpful comments during the development of the work contained here. I am particularly indebted to her for showing me how the Green-Schwarz cancellation of the  $U(1)_X$  anomaly works in the linear multiplet formulation and have borrowed heavily from written communications with her in preparing the Appendix. I would also like to thank Brent Nelson for useful comments. This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797.

## Appendix

### A Charge normalization

In the FIQS model, unconventional normalizations for the  $U(1)$  charges have been chosen to keep the tables of charges simple and amenable to com-

puter assisted analysis. The generator  $Q_X^I$  acting on the  $E_8 \times E_8$  root torus is given by

$$Q_X = 6(0, 0, 0, 0, 0, 0, 0, 0; 1, -1, 1, 0, 0, 0, 0, 0). \quad (\text{A.1})$$

The affine level of a  $U(1)$  group may be defined [44] as

$$k_Q = 2 \sum_{I=1}^{16} (Q^I)^2. \quad (\text{A.2})$$

With this convention, the FIQS normalization gives  $k_{Q_X} = 6^3$ . To go to a normalization where the coupling constant for the  $U(1)_X$  group will be the universal coupling at the string scale, we must rescale the generator  $Q_X \rightarrow Q'_X$  so that  $k_{Q'_X} = 1$ . Then

$$Q'_X = \frac{1}{6\sqrt{6}} Q_X. \quad (\text{A.3})$$

Since the original normalization satisfied<sup>13</sup>

$$\text{tr } Q_X^3 = 27 \cdot \text{tr } Q_X = 27 \cdot 24 \cdot \text{tr } T^a T^a Q_X = 27 \cdot 24 \cdot 54, \quad (\text{A.4})$$

it can be checked that the rescaled generator satisfies

$$24 \text{tr } (T^a T^a Q'_X) = 8 \text{tr } Q'^3_X = \text{tr } Q'_X = 36\sqrt{6}, \quad (\text{A.5})$$

as required by anomaly matching [44]. Indeed, if the  $U(1)_X$  vector superfield  $V_X$  is shifted by  $\delta V_X = (1/2)(\Lambda + \bar{\Lambda})$ , then the resulting anomalous transformation of the lagrangian is

$$\delta \mathcal{L} = \frac{1}{16\pi^2} \sum_a \text{tr } (T^a T^a Q'_X) \left[ \text{Re } \lambda F^a \cdot F^a + \text{Im } \lambda F^a \cdot \tilde{F}^a \right] + \dots \quad (\text{A.6})$$

where  $\lambda = \Lambda|$ . We introduce our counterterm<sup>14</sup> as

$$\mathcal{L}_{GS, V_X} = \delta_X \int EVV_X \quad (\text{A.7})$$

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<sup>13</sup>The nonabelian generators  $T^a$  are normalized such that  $\text{tr } T^a T^a = 1/2$  for a fundamental representation of  $SU(N)$ .

<sup>14</sup>We work in Kähler superspace [45] and use the linear multiplet formulation where  $V$  is a real superfield which satisfies *modified linearity conditions* and contains the dilaton  $\ell$  as its lowest component [46, 12].

from which it follows that under the shift in  $V_X$

$$\delta\mathcal{L}_{GS,V_X} = \frac{\delta_X}{2} \int EV(\Lambda + \bar{\Lambda}) \quad (\text{A.8})$$

which when we go to components yields

$$\delta\mathcal{L}_{GS,V_X} = -\frac{\delta_X}{8} \sum_a \left( \text{Re } \lambda F^a \cdot F^a + \text{Im } \lambda F^a \cdot \tilde{F}^a \right) + \dots \quad (\text{A.9})$$

The anomaly is cancelled if we choose

$$\delta_X = \frac{1}{2\pi^2} \text{tr } T^a T^a Q'_X. \quad (\text{A.10})$$

When combined with other terms in the lagrangian, the component form of (A.7) gives

$$D_X = \sum_i q_X^i \hat{K}_i \phi^i + \frac{\delta_X}{2} \ell \equiv \sum_i q_X^i \hat{K}_i \phi^i + \xi. \quad (\text{A.11})$$

From this, we see that the FI term  $\xi$  is given by

$$\xi = (2\ell) \frac{\delta_X}{4} = \frac{2\ell}{8\pi^2} \text{tr } T^a T^a Q'_X. \quad (\text{A.12})$$

With the  $U(1)_X$  generator chosen such that  $k_{Q'_X} = 1$ , equation (A.5) gives

$$\xi = \frac{2\ell}{192\pi^2} \text{tr } Q'_X, \quad (\text{A.13})$$

which may be recognized as the form typically quoted in the literature once it is realized that if we neglect nonperturbative corrections to the Kähler potential of the dilaton  $\ell$ , the universal coupling constant at the string scale is given by  $g^2 = 2\ell$ . In the FIQS normalization, the FI term is given by

$$\xi = \frac{2\ell}{192\pi^2} \frac{1}{6\sqrt{6}} \text{tr } Q_X \quad (\text{A.14})$$

which gives a significantly smaller number than if we had not accounted for the unconventional normalization of the  $U(1)_X$  charge. In the FIQS model  $\text{tr } Q_X = 1296$ , yielding

$$\xi \approx 2\ell \times 4.7 \times 10^{-2} \sim 5 \times 10^{-2}, \quad (\text{A.15})$$

where we have used  $2\ell \approx g^2$ , and  $0.5 \lesssim g^2 \lesssim 1$ . The scale of  $U(1)_X$  breaking is given by  $\Lambda_X \sim \sqrt{\xi} \sim 0.22 m_P \approx 5 \times 10^{17} \text{ GeV} \approx \Lambda_s$ , the string scale.

One must also take proper account of charge normalization for the SM hypercharge, as was pointed out in ref. [14]. For example, in the FIQS model  $k_Y = 11/3$ . Then the charge generator which will have the unified coupling at the string scale is  $Y' = \sqrt{3/11} Y$ . This is to be compared with the  $G_{GUT} \supseteq SU(5)$  relative factor of  $\sqrt{3/5}$ . Thus, the boundary value of the properly normalized hypercharge coupling  $g'$  at the electroweak scale in the FIQS model is related to the one usually used in GUT-inspired renormalization group evolution of the couplings in the MSSM by

$$g'(\text{FIQS})|_{M_Z} = \sqrt{11/5} \ g'(\text{MSSM})|_{M_Z}. \quad (\text{A.16})$$

This clearly does violence to unification of the couplings. In short, it is necessary to include hypercharge normalization among the criteria to be checked when searching for viable string-derived models.

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